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Non-linear conductivity and quantum interference in disordered metals

M. Leadbeater[1], R. Raimondi[1], P. Schwab[2,3,a], and C. Castellani[2]

[1] INFM e Dipartimento di Fisica "E. Amaldi", Università di Roma Tre, Via della Vasca Navale 84, 00146 Roma, Italy
[2] INFM e Dipartimento di Fisica, Università "La Sapienza", 00185 Roma, Italy
[3] Institut für Physik, Universität Augsburg, 86135 Augsburg, Germany

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Abstract. We report on the non-linear electric field effect in the conductivity of disordered conductors. We find that the electron-electron interaction in the particle-hole triplet channel strongly affects the non-linear conductivity. The non-linear effect introduces a field dependent temperature scale T_E and provides a microscopic mechanism for electric field scaling at the metal-insulator transition. We also study the magnetic field dependence of the non-linear conductivity and suggest possible ways to experimentally verify our predictions. These effects offer a new probe to test the role of quantum interference at the metal-insulator transition in disordered conductors.

PACS. 72.10.-d Theory of electronic transport; scattering mechanisms – 72.15.Rn Localization effects (Anderson or weak localization)

Disordered conductors have been the subject of theoretical and experimental study for almost twenty years [1,2]. Recently there has been a strong resurgence of interest in the field due to the unexpected discovery of a metal-insulator transition (MIT) in two-dimensional systems [3]. Various suggestions have been made concerning the origin of the temperature dependence of the resistivity in the metallic phase and the nature of the metal-insulator transition [4]. One main question is whether the transition is of a classical origin or if it is a real quantum phase transition. In the first case, if a standard Landau quasi-particle picture applies the observed resistivity could be attributed to a temperature dependent scattering time in the context of the semi-classical Boltzmann-Landau kinetic equation [5]. In the second case, it has been pointed out [6] that the occurrence of a metallic phase and a metal-insulator transition in two dimensional systems is indeed possible within the standard theory of disordered-interacting electrons [7].

To discriminate between these possibilities one needs specific probes for quantum interference effects. Magnetoresistance measurements are the standard probe for the amplitude of weak localization (WL) and for quantum interference from the combined contribution of disorder and electron-electron interaction (EEI) in the particle-hole triplet channel [8]. Until now no probe has been known for the particle-hole singlet channel. In this paper we propose a new probe for the EEI contribution (both for the singlet and the triplet) based on the non-linear conductivity in the presence of a static electric field. We recall that WL is not affected by such a field [1,9].

In reference [10] we have found that a static (or low frequency) electric field acts as a source of dephasing in the particle-hole channel and introduces a characteristic temperature $T_E = (De^2E^2)^{\frac{1}{3}}$ below which interference effects are suppressed. Here we extend this result in two important directions. First, we include the scattering amplitude in the particle-hole triplet channel, γ_2, which is large in the metallic phase of two-dimensional electron systems [11]. We find that the value of γ_2 has a dramatic effect on the non-linear conductivity at large electric field. Second, we allow for the renormalization of the quasi-particle diffusion coefficient $D \to D_{\mathrm{qp}}$, defined below, and discuss its implications for the temperature scale T_E and for electric field scaling.

Before giving details of the mathematical derivation a qualitative understanding of the effect may be obtained by simple physical arguments along the lines of reference [12]. In a generalised Hartree-Fock picture one electron is scattered by the potential created by all the other electrons. From a semi-classical point-of-view, a local, single-particle quantity, like current, only involves closed paths. Furthermore, the EEI corrections are dominated by all the other electrons retracing backwards-in-time (as holes) almost the same closed paths. At finite temperature only trajectories which are traversed in a time $\eta < 1/T$ contribute to quantum corrections. Although the two electrons go around the same closed path they have different starting positions. The first electron starts at the observation point $\mathbf{x}_1$ at time zero, while the second electron will only start

[a] e-mail: peter.schwab@physik.uni-augsburg.de

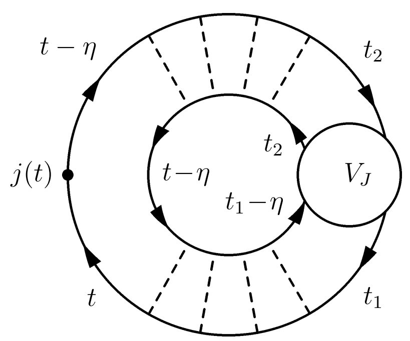

Fig. 1. Pictorial representation of the current formula. Four dashed lines represent a diffuson.

to retrace the path at the point of interaction $\mathbf{x}_2$ at time t_1. This means that the second electron will traverse part of the closed path at a different time. In the presence of a vector potential the accumulated phase difference is then $\phi_1 - \phi_2 = e\int_{t_1-\eta}^{0} \mathrm{d}t'\dot{\mathbf{x}}_1(t')\cdot\mathbf{A}(\mathbf{x}_1(t'),t') - e\int_{t_1}^{\eta}\mathrm{d}t'\dot{\mathbf{x}}_2(t')\cdot\mathbf{A}(\mathbf{x}_2(t'),t')$. If the vector potential is time independent (*e.g.* a magnetic field) these phases completely cancel. However, if the vector potential is time dependent (as for a static electric field) the time delay leads to a finite phase difference $\phi_1-\phi_2 = e(\mathbf{x}_2-\mathbf{x}_1)\cdot\mathbf{E}\eta$, which suggests that the EEI correction should be sensitive to a static electric field, in contrast to WL. Such a phase-sensitivity leads to non-linear conductivity. It is possible to estimate the typical electric field scale where dephasing and non-linear effects in a weakly disordered metal become important. The typical time scale is the inverse temperature and the typical length scale is the thermal length $L_{\mathrm{T}} = (D_{\mathrm{qp}}/T)^{1/2}$. The non-linear effects become important when the phase difference induced by the electric field is of order one, which leads to the condition that the voltage drop over a thermal length becomes of the order of the temperature, *i.e.*, when $eEL_{\mathrm{T}} \sim T$. This condition defines the temperature scale T_E given above.

While the above physical discussion is quite general, we now present a quantitative theory which relies upon the weak disorder limit, $g(L_{\mathrm{T}}) \gg 1$, g being the conductance at scale L_{T}. We start with the expression for the EEI quantum correction to the current due to the interplay between disorder and interaction. Within the real-time Keldysh formalism [13] we obtain:

$$\delta\mathbf{j}(t) = -\frac{4\tau^2 e}{\pi}\int \mathrm{d}\eta\mathrm{d}t_1\mathrm{d}t_2\left(\frac{\pi T}{\sinh(\pi T\eta)}\right)^2\sum_q D_{\mathrm{qp}}\mathbf{q}$$
$$\times\sum_{J,M} V_{J,M}(\mathbf{q},t_1-t_2)D_{J,M}^{\eta'=0}(t_2,t-\eta;\mathbf{q})$$
$$\times D_{J,M}^{\eta}(t-\eta/2,t_1-\eta/2;\mathbf{q}). \quad (1)$$

A pictorial representation of this equation is shown in Figure 1. The details of its derivation may be found in [10]. The sum $\sum_{J,M}$ is over one singlet ($J=0, M=0$) and three triplet channels with $J=1, M=0,\pm1$. In equation (1) the propagation is in the presence of an external vector potential and the short time cut-off in the problem is the elastic scattering time τ. $V_{J,M}$ and $D^{\eta}_{J,M}(t,t')$ are the interaction and the diffusion propagator in the spin channel (J,M). Here the time arguments t, t' refer to the incoming and outgoing centre-of-mass time of the particle-hole pair and η to the relative time which is constant during the propagation. Notice that both $V_{J,M}$ and $D_{J,M}$ are retarded functions. The factor containing the hyperbolic sine comes from the Fourier transform of Fermi functions and limits us to trajectories with traverse time $\eta < 1/T$. The interaction is found by summing ladder diagrams and is given by

$$V_{J,M}(\mathbf{q},\omega) = \gamma_J\frac{-\mathrm{i}\omega + D_{\mathrm{qp}}q^2 + \mathrm{i}M\tilde{\Omega}_s}{-\mathrm{i}(1-2\gamma_J)\omega + D_{\mathrm{qp}}q^2 + \mathrm{i}M\tilde{\Omega}_s}, \quad (2)$$

where γ_J is the static amplitude for scattering between particles and holes. The quasi-particle diffusion constant can be expressed in terms of the particle diffusion constant $D_{\mathrm{qp}} = D/Z$. The parameter Z, which arises in the context of the Fermi liquid theory of disordered systems [7] as the energy renormalization, plays the role of mass renormalization, m^*/m, in the effective Fermi liquid theory of disordered systems [14]. The interaction amplitude in the spin singlet channel is given by $\gamma_{J=0} = 1/2$ for long range Coulomb forces. The triplet amplitude, for which we adopt in the following the standard notation $\gamma_{J=1} = -\gamma_2/2$, is related to the Landau parameter F_a^0 *via* $\gamma_2 = -A_a^0 = -F_a^0/(1+F_a^o)$. The diffusion propagator $D_{J,M}$ is given by the solution of the differential equation

$$\left\{\frac{\partial}{\partial t} + D_{\mathrm{qp}}\left[-\mathrm{i}\nabla + e\mathbf{A}_\eta(\mathbf{r},t)\right]^2 + \mathrm{i}M\tilde{\Omega}_s\right\}D^{\eta}_{J,M}(t,t')$$
$$= \frac{1}{\tau}\delta(t-t')\delta(\mathbf{r}-\mathbf{r}'). \quad (3)$$

where $\mathbf{A}_\eta(\mathbf{r},t) = \mathbf{A}(\mathbf{r},t+\eta/2) - \mathbf{A}(\mathbf{r},t-\eta/2)$. The term $\mathrm{i}M\tilde{\Omega}_s$ is due to the Zeeman coupling, where $\tilde{\Omega}_s = (1+\gamma_2)\Omega_s$ with $\Omega_s = g\mu_{\mathrm{B}}H$.

We now evaluate the current explicitly. According to equation (3) the interaction $V_{J,M}$ and the first of the two diffusons in (1) do not depend on the vector potential. An electric field, however, affects the second diffuson in (1) due to the non-zero time delay η between the particle and hole. For a static field the vector potential is $\mathbf{A}(t) = -\mathbf{E}t$ and the solution of (3) is

$$D^{\eta}_{J,M}\left(t-\frac{\eta}{2},t_1-\frac{\eta}{2};\mathbf{q}\right) =$$
$$\frac{1}{\tau}\exp\left\{-\left[D_{\mathrm{qp}}(\mathbf{q}-e\mathbf{E}\eta)^2 - \mathrm{i}M\tilde{\Omega}_s\right](t-t_1)\right\}.$$

Table 1. Table of coefficients which appear in the expressions for the current. For small γ_2 these reduce to $f_d^1(\gamma_2) = 2/d - 3\gamma_2/2$, $f_d^3(\gamma_2) = 4/(d(2+d)) - \gamma_2/4$, $g_d^1(\gamma_2) = \gamma_2/2$ and $g_d^3(\gamma_2) = -\gamma_2/12$.

$f_d^1(\gamma_2)$	$\frac{2}{d} - 3\frac{4(1+\gamma_2)^{\frac{d}{2}}-4-2d\gamma_2}{(d-2)d\gamma_2}$
$f_d^3(\gamma_2)$	$\frac{4}{d(d+2)}\left(1+3\frac{[24+(16-4d)\gamma_2](1+\gamma_2)^{\frac{d}{2}}-24-(2+d)\gamma_2(8+d\gamma_2)}{(d-4)(d-2)\gamma_2^3}\right)$
$f_2^1(\gamma_2)$	$1+3\left[1-\frac{1+\gamma_2}{\gamma_2}\ln(1+\gamma_2)\right]$
$f_2^3(\gamma_2)$	$\frac{1}{2}+\frac{3}{2}\left[\frac{6+5\gamma_2}{\gamma_2^2}-\frac{(6+2\gamma_2)(1+\gamma_2)}{\gamma_2^3}\ln(1+\gamma_2)\right]$
$g_d^1(\gamma_2)$	$2\frac{2(1+\gamma_2)^{\frac{d}{2}}-(1+\gamma_2)^2[2+(d-4)\gamma_2]}{(d-6)(d-4)\gamma_2}$
$g_d^3(\gamma_2)$	$4\frac{[24+4(8-d)\gamma_2](1+\gamma_2)^{\frac{d}{2}}-\{24+(d-2)\gamma_2[8+(d-4)\gamma_2]\}(1+\gamma_2)^2}{(d-8)(d-6)(d-4)(d-2)\gamma_2^3}$
$g_2^1(\gamma_2)$	$\frac{1}{2}\gamma_2(1+\gamma_2)$
$g_2^3(\gamma_2)$	$\frac{1+\gamma_2}{\gamma_2^2}\left[\frac{3\gamma_2+6-\gamma_2^2}{6}-\frac{1+\gamma_2}{\gamma_2}\ln(1+\gamma_2)\right]$

The equation for the current after integrating over the momentum then becomes

$$\delta\mathbf{j}_{J,M} = -\mathbf{E}\frac{4e^2D_{\rm qp}}{\pi}\gamma_J\left(\frac{1-2\gamma_J}{4\pi D_{\rm qp}}\right)^{d/2}$$
$$\times\int_\tau^\infty \mathrm{d}\eta\left[\frac{\pi T}{\sinh(\pi T\eta)}\right]^2\int_0^\eta \mathrm{d}t_1\frac{t_1\eta}{(\eta-2\gamma_J t_1)^{1+d/2}}$$
$$\times\cos[M\Omega_s(\eta-2\gamma_J t_1)]$$
$$\times\exp\left[-T_E^3\eta^2 t_1(\eta-t_1)/(\eta-2\gamma_J t_1)\right] \qquad (4)$$

where we have introduced $T_E^3 = D_{\rm qp}(eE)^2$ and d is the dimension. It is clear from this equation that the electric field provides a dephasing time $\sim T_E^{-1}$, since in the low temperature limit $T \ll T_E$ the exponential now cuts off all times larger than T_E^{-1}.

We first consider the current in the weak electric field regime and derive the leading non-linear terms. In the absence of magnetic field we find

$$\delta\mathbf{j} = \mathbf{E}\frac{e^2}{2^{d-1}\pi^2}\left(\frac{D_{\rm qp}}{T}\right)^{\frac{2-d}{2}}\int_{\pi T\tau}^\infty \mathrm{d}x\frac{x^{2-\frac{d}{2}}}{\sinh^2(x)}$$
$$\times\left(f_d^1(\gamma_2)+f_d^3(\gamma_2)\frac{x^3T_E^3}{\pi^3T^3}\right) \qquad (5)$$

where the functions $f_d^{1,3}$ are listed in Table 1. For the sake of completeness we have also included the term linear in the electric field which reproduces the well-known interaction correction to the conductivity. Notice that the functions $f_d^{1,3}(\gamma_2)$ are the sum of the singlet and the triplet contributions. The results for $\delta\sigma = \delta|\mathbf{j}|/|\mathbf{E}|$ are

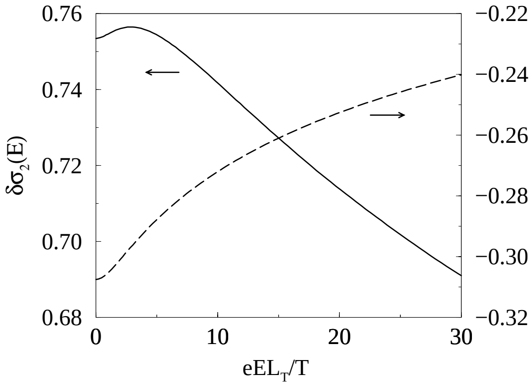

Fig. 2. The electric field dependence of the interaction correction to the conductivity in two dimensions in units of $e^2/\hbar$ for $\gamma_2 = 0$ (dashed line) and $\gamma_2 = 5$ (solid line). Note the different scales used for the two values of γ_2.

$$\delta\sigma_1 = \frac{e^2}{\pi^2}L_{\rm T}\left[-2.46f_1^1(\gamma_2)+\frac{4.88}{\pi^3}f_1^3(\gamma_2)\frac{T_E^3}{T^3}\right]$$
$$\delta\sigma_2 = \frac{e^2}{2\pi^2}\left[-f_2^1(\gamma_2)\ln\left(\frac{\rm e}{2\pi T\tau}\right)+\frac{\pi}{30}f_2^3(\gamma_2)\frac{T_E^3}{T^3}\right]$$
$$\delta\sigma_3 = \frac{e^2}{4\pi^2}L_{\rm T}^{-1}\left[1.83f_3^1(\gamma_2)+\frac{2.32}{\pi^3}f_3^3(\gamma_2)\frac{T_E^3}{T^3}\right] \qquad (6)$$

where $L_{\rm T}$ is the thermal length defined previously and we have left out temperature independent terms. We recall that, in the case of spin-singlet interactions only ($\gamma_2 = 0$) the conductivity decreases with decreasing temperature (*i.e.* $f_d^1(0) > 0$), whereas for sufficiently large triplet amplitude γ_2 the latter dominates and leads to an increase of conductivity with decreasing temperature (*i.e.* $f_d^1(\gamma_2) < 0$). The non-linear coefficient $f_d^3(\gamma_2)$, however, is generically positive and only changes sign for large γ_2 in $d = 3$.

We study the cross-over behaviour from small to large electric fields by numerically integrating equation (4). The conductivity as a function of the electric field is plotted in Figure 2 for two values of γ_2 for $d = 2$. At zero field, for $\gamma_2 = 0$ ($\gamma_2 = 5$) the correction is localising (anti-localising) with $\delta\sigma_2 < 0$ ($\delta\sigma_2 > 0$). The quadratic increase at small fields has a positive curvature irrespective of the value of γ_2. At large field, the temperature scale disappears and the correction $\delta\sigma$ has the same form as the linear conductivity with T_E replacing the temperature.

Non-linear effects also appear in the magnetoconductance which originate from the magnetic field depression of the $M = \pm 1$ triplet contributions to the current.

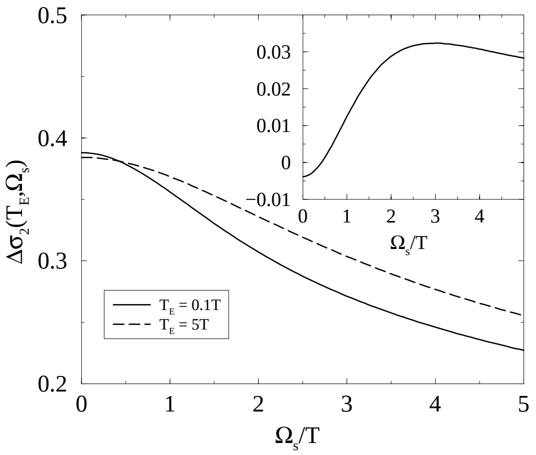

Fig. 3. The magnetic field dependence of the conductivity in two dimensions in units of $e^2/\hbar$ for two different values of T_E and for $\gamma_2 = 2.5$. The inset shows the difference $\Delta\sigma_2(T_E = 5T) - \Delta\sigma_2(T_E = 0.1T)$ of the two curves.

In particular we find for small T_E and small Zeeman energy Ω_s

$$\Delta\sigma_1 = -\frac{e^2}{\pi^2} L_{\mathrm{T}} \frac{\Omega_s^2}{T^2} \left[\frac{2.32}{\pi^2} g_1^1(\gamma_2) + \frac{41.85}{\pi^5} g_1^3(\gamma_2) \frac{T_E^3}{T^3} \right]$$
$$\Delta\sigma_2 = -\frac{e^2}{2\pi^2} \frac{\Omega_s^2}{T^2} \left[\frac{3\zeta(3)}{2\pi^2} g_2^1(\gamma_2) + \frac{\pi}{42} g_2^3(\gamma_2) \frac{T_E^3}{T^3} \right]$$
$$\Delta\sigma_3 = -\frac{e^2}{4\pi^2} L_{\mathrm{T}}^{-1} \frac{\Omega_s^2}{T^2} \left[\frac{1.58}{\pi^2} g_3^1(\gamma_2) + \frac{13.04}{\pi^5} g_3^3(\gamma_2) \frac{T_E^3}{T^3} \right] \tag{7}$$

where $\Delta\sigma = \sigma(\Omega_s) - \sigma(0)$ and the functions $g_d^{1,3}$ are also shown in Table 1 [15]. To illustrate the behaviour at large Ω_s we again resort to numerical integration of equation (4). In Figure 3 we show the magnetic field dependence of the current for two choices of T_E for $\gamma_2 = 2.5$. Notice that for such a value of γ_2 the zero magnetic field conductivity interference correction has an anti-localizing character. This explains the suppression of conductivity with increasing T_E. For small T_E we obtained the standard behaviour of a initial quadratic decrease on the scale of the temperature followed by a logarithmic suppression of the corrections. For large T_E however, the temperature disappears as an energy scale and, although the curve appears similar, the scale of the magnetic field is now set by T_E. Expanding equation (4) to leading order in the magnetic field for $T_E \gg T$ one obtains $\Delta\sigma \propto \Omega_s^2/T_E^2$.

The effects described in this paper may be detected by measuring the current-voltage characteristics. In such a measurement however the electron temperature changes with the applied voltage and one has to discriminate heating from non-heating non-linear effects. A direct way to isolate the non-linear contribution due to the dephasing effect of E would be to measure the electron temperature T_{el} for a given E (for instance by noise measurements as in [16]). Then $\sigma(T_{\mathrm{el}}, 0) - \sigma(T, E)$ yields the effect of the electric field on the EEI contribution and provides a direct probe of the relevance of quantum interference in the p-h channels. Alternatively, at low temperature, where $T_{\mathrm{el}}\tau_{\mathrm{el-ph}} \gg 1$ ($\tau_{\mathrm{el-ph}}$ is the heat electron-phonon relaxation time) non-heating effects could be detected by exploiting the different time scales $\tau_{\mathrm{el-ph}}$ and T_{el}^{-1} which control the frequency dependence of heating and non-heating effects respectively. In a time-dependent electric field, $E(t) = E\cos(\Omega t)$, the electron temperature becomes time dependent. For frequencies $\Omega > 1/\tau_{\mathrm{el-ph}}$ however the temperature cannot follow the electric field, *i.e.* heating becomes time independent. Non-linearities due to quantum interference on the other hand follow the electric field instantly as long as the frequency remains smaller than the temperature. Thus measuring non-linear response in the presence of a microwave with a frequency of the order $\Omega \geq 1/\tau_{\mathrm{el-ph}}$ offers a possibility to detect the predicted effects [17].

Possible non-heating effects have already been observed in different materials [18–21]. In particular a remarkable electric field scaling was reported near the metal-insulator transition [19]. Although the critical regime of the MIT is most likely out of the reach of our perturbative analysis around the metallic state we would like to remark that our non-linear effect provides an explicit mechanism for the electric field scaling *via* the temperature scale T_E. On the basis of general scaling arguments close to a quantum critical point, the temperature scales as $T \sim \xi^{-z}$ where ξ is the correlation length and z is the dynamical critical exponent [22]. In a diffusive system temperature and length scales are related *via* the diffusion constant with $T \sim D_{\mathrm{qp}}(\xi)/\xi^2$ implying a scaling of $D/Z = D_{\mathrm{qp}}$ near the critical point as $D_{\mathrm{qp}} \sim \xi^{2-z}$. From our relation $T_E^3 = D_{\mathrm{qp}} e^2 E^2$ one then obtains the scaling [19] $E \sim \xi^{-(1+z)}$. In the experiments z is near one, which corresponds to a vanishing quasi-particle density of states and a growing quasi-particle diffusion constant near the transition. This small value of $z < 2$ implies that the electronic specific heat would vanish as $c_v \sim T\xi^{z-2} \sim T^{2/z}$. From these considerations one expects large non-linear effects in the low temperature phase near the critical point.

Such large effects have been observed in reference [20] for a GaAs metallic sample near the metal-insulator transition. At low temperature ($T = 80$ mK) the conductivity shows a non-monotonic behaviour of the type shown in Figure 2 for large γ_2. Notice that the initial weak increase of the conductivity cannot be explained in terms of heating. By comparing with this experiment it appears that the electric field scale over which the effect is observed is at least two orders of magnitude below what we would predict based upon a naive estimate of the diffusion constant from the conductivity $\sigma = 2e^2DN_0$ and the free particle density of states N_0 (*i.e.* assuming $D_{\mathrm{qp}} \sim D$) [23]. However, by allowing for quasi-particle diffusion constant renormalization as implied by scaling one can obtain larger effects, even though at present we cannot claim a definitive agreement with the experiments [20].

Finally, we would like to point out that the effects discussed in this paper should have an enhanced relevance in the presence of strong local electric fields such as in percolative metallic systems. Whether percolation is important near the observed MIT is an open issue.

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References

1. B.L. Altshuler, A.G. Aronov, D.E. Khmelnitskii, A.G. Larkin, in *Quantum Theory of Solids*, edited by I.M. Lifshitz (MIR Publishers, Moscow, 1982); B.L. Altshuler, A.G. Aronov, in *Electron-Electron Interactions in Disordered Systems*, edited by M. Pollak, A.L. Efros (North-Holland, Amsterdam, 1985), p. 1.
2. P.A. Lee, T.V. Ramakrishnan, Rev. Mod. Phys. **57**, 287 (1985); D. Belitz, T.R. Kirkpatrick, Rev. Mod. Phys. **66**, 261 (1994).
3. S.V. Kravchenko, W.E. Mason, G.E. Bower, J.E. Furneaux, V.M. Pudalov, M. D'Iorio, Phys. Rev. B **51**, 7038 (1995).
4. V. Dobrosavljevic, E. Abrahams, E. Miranda, S. Chakravarty, Phys. Rev. Lett. **79**, 455 (1997); P. Phillips *et al.*, Nature **395**, 253 (1998); S. Chakravarty *et al.*, Phil. Mag. B **79**, 859 (1999); V.M. Pudalov, JETP Lett. **66**, 175 (1997); D. Belitz, T.R. Kirkpatrick, Phys. Rev. B **58**, 8214 (1998); Q. Si, C.M. Varma, Phys. Rev. Lett. **81**, 4951 (1998).
5. B.L. Altshuler, D.L. Maslov, Phys. Rev. Lett. **82**, 145 (1999); T.M. Klapwijk, S. Das Sarma, Solid State Commun. **110**, 581 (1999); S. Das Sarma, E.H. Hwang, Phys. Rev. Lett. **83**, 164 (1999).
6. C. Castellani, C. Di Castro, P.A. Lee, Phys. Rev. B **57**, R9381 (1998).
7. A.M. Finkel'stein, JETP **57**, 97 (1983); C. Castellani, C. Di Castro, P.A. Lee, M. Ma, Phys. Rev. B **30**, 527 (1984).
8. B.L. Altshuler, A.G. Aronov, P.A. Lee, Phys. Rev. Lett. **44**, 1288 (1980); B.L. Altshuler, D. Khmel'nitzkii, A.I. Larkin, P.A. Lee, Phys. Rev. B **22**, 5141 (1980).
9. G. Bergmann, Z. Phys. B **49**, 133 (1982).
10. R. Raimondi, P. Schwab, C. Castellani, Phys. Rev. B **60**, 5818 (1999). See also this reference for a discussion of non-linearity in the context of the saturation of the dephasing time.
11. P.T. Coleridge, A.S. Sachrajda, P. Zawadzki, `cond-mat/9912041`; D. Simonian, S.V. Kravchenko, M.P. Sarachik, V.M. Pudalov, Phys. Rev. B **57**, R9420 (1998).
12. I.L. Aleiner, B.L. Altshuler, E. Gershenson, Waves Random Media **9**, 201 (1999).
13. For recent applications of the Keldysh formalism to disordered conductors see A. Kamenev, A. Andreev, Phys. Rev. B **60**, 2218 (1999); C. Chamon, A.W.W. Ludwig, C. Nayak, Phys. Rev. B **60**, 2239 (1999).
14. C. Castellani, C. Di Castro, Phys. Rev. B **34**, 5935 (1986); C. Castellani, G. Kotliar, P.A. Lee, Phys. Rev. Lett. **59**, 323 (1987).
15. Note that the numerical factor we obtain in front of the $g_2^1(\gamma_2)$ term is slightly larger (about 10%) than the factor given in the literature.
16. R. de-Picciotto *et al.*, Nature **389**, 162 (1997).
17. S.A. Vitkalov (private communication).
18. G. Bergmann, Wei Wei, Yao Zou, R.M. Mueller, Phys. Rev. B **41**, 7386 (1990); J. Liu, N. Giordano, Phys. Rev. B **43**, 1385 (1991).
19. S.V. Kravchenko, D. Simonian, M.P. Sarachik, W. Meson, G.E. Fourneaux, Phys. Rev. Lett. **77**, 4938 (1996), R. Heemskerk, T.M. Klapwijk, Phys. Rev. B **58**, R1754 (1998).
20. J. Yoon, C.C. Li, D. Shahar, D.C. Tsui, M. Shayegan, Phys. Rev. Lett. **82**, 1744 (1999).
21. V. Senz, *et al.*, Annalen Phys. **8**, Special Issue, 237 (1999).
22. For a recent review see S.L. Sondhi, S.M. Girvin, J.P Carini, D. Shahar, Rev. Mod. Phys. **69**, 315 (1997).
23. Note that in this estimate we assumed a uniform electric field throughout the sample.